

RETROGRADE CONDENSATION IN THE CASE OF STEADY RADIAL FILTRATION

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A theoretical analysis of the processes occurring during filtration close to a well of a gas-condensate field is presented.

In the practice of the exploitation of gas-condensate fields of complex composition, a situation is often encountered where the bed mixture occurs in a gas phase under initial thermobaric conditions and then becomes thermodynamically unstable when the bed pressure decreases as a result of gas withdrawal. The so-called phenomenon of retrograde condensation [1] takes place, where a liquid phase comes out of the gas on decrease in pressure. Since the viscosity of the liquid greatly exceeds that of the gas, retrograde condensation exerts a strong effect on filtration processes. In particular, it is well known that the fallout of condensate can decrease by orders of magnitude the yield of an exploratory well with depression remaining intact. Therefore, we are confronted with the problem of predicting the dynamics of a bed mixture with allowance for gas-liquid phase transitions. In principle, along with the solution of a system of partial differential equations that describe the filtration of a multicomponent mixture, such a problem presupposes the calculation of phase equilibria at each point [2]. The calculation is a rather complex problem in itself and requires much time if performed on a computer. In view of this, of special interest are cases where the thermodynamic part of the problem can be simplified without loss of generality.

In the present work a steady-state two-phase flow near a well is investigated. It is shown that the system of equations of two-phase multicomponent filtration is reduced to ordinary differential equations admitting analytical solutions under certain natural assumptions. To obtain an adequate description of phase conversions, it is sufficient to know the thermodynamic behavior of a mixture having a single fixed composition.

Suppose there is a steady-state cylindrical symmetric flow of a multicomponent mixture in a homogeneous isotropic porous medium with the permeability k . We consider an isothermal process, and let T be the corresponding temperature. Let us introduce the geometric parameters of the problem: r , distance to the axis of the well; r_w , radius of the well along the drill bit; h , thickness of the permeable layer. Let N be the number of components in the bed mixture. Then the equations of multicomponent two-phase filtration [2] are reduced to the conservation equations of the flows of the components

$$\frac{d}{dr} \left(rk \left(n_{ig} f_g \mu_g^{-1} \frac{dp_g}{dr} + n_{i \text{ liq}} f_{liq} \mu_{liq}^{-1} \frac{dp_{liq}}{dr} \right) \right) = 0, \quad (1)$$

$$i = 1, \dots, N.$$

Here n_{ig} is the density of the i -th component molecules in the liquid phase, μ_g is the viscosity of the gas phase, μ_{liq} is the viscosity of the liquid phase, p_g is the pressure in the gas phase, p_{liq} is the pressure in the liquid phase, f_g and f_{liq} are the phase permeabilities for the gas and the liquid, respectively, which are functions of pore space saturation s with the liquid phase. Let us introduce the notation $\eta = \ln (r/r_w)$.

Equations (1) have the first integrals

$$n_{ig} f_g \mu_g^{-1} \frac{dp_g}{d\eta} + n_{i \text{ liq}} f_{\text{liq}} \mu_{\text{liq}}^{-1} \frac{dp_{\text{liq}}}{d\eta} = Q_i / (2\pi h k), \quad (2)$$

$$i = 1, \dots, N,$$

where Q_i is the flux of the molecules of the i -th component. If it is assumed that the expressions for f_g and f_{liq} as functions of s , the expressions for the viscosities $\mu_g = \mu(T, n_{ig})$, $\mu_{\text{liq}} = \mu_{\text{liq}}(T, n_{i \text{ liq}})$, and the equations of state $p_g = p(T, n_{ig})$, $p_{\text{liq}} = p(T, n_{i \text{ liq}})$ are known, then we seek the $(2N + 1)$ unknown functions n_{ig} , $n_{i \text{ liq}}$, s . The problem becomes closed when Eqs. (2) are supplemented with the phase equilibrium equations

$$\kappa_i(T, n_{j \text{ liq}}) = \kappa_i(T, n_{jg}), \quad i, j = 1, \dots, N, \quad (3)$$

where κ_i are the chemical potentials, and also with the equation for the capillary pressure jump

$$p_g - p_{\text{liq}} = p_{\text{jump}}, \quad (4)$$

where $p_{\text{jump}} = p_{\text{jump}}(s)$ is a prescribed function.

Let us investigate Eqs. (2) in more detail. Let us introduce the notation: $n_g = \sum_{i=1}^N n_{ig}$, the total density of the gas phase; $n_{\text{liq}} = \sum_{i=1}^N n_{i \text{ liq}}$, the total density of the liquid phase; $Q = \sum_{i=1}^N Q_i$, the total flow of the mixture molecules; $c_{ig} = n_{ig}/n_g$ is the concentration of the components in the gas phase; $c_{i \text{ liq}} = n_{i \text{ liq}}/n_{\text{liq}}$ is the concentration of the components in the liquid phase; w is the mole fraction of the liquid phase in the mixture; $q = Q/(2\pi k h)$; $C_g = n_g f_g \mu_g^{-1} q^{-1}$, $C_{\text{liq}} = n_{\text{liq}} f_{\text{liq}} \mu_{\text{liq}}^{-1} q^{-1}$. Then the system of equations (2) can be rearranged into the form

$$C_g \frac{dp_g}{d\eta} c_{ig} + C_{\text{liq}} \frac{dp_{\text{liq}}}{d\eta} c_{i \text{ liq}} = Q_i / Q. \quad (5)$$

Assuming that far from the well at pressures close to the initial bed pressure p_0 , the mixture occurs in the gas phase with the concentrations of components c_{i0} , it can easily be seen that $Q_i/Q = c_{i0}$. Thus Eqs. (5) can be interpreted as conditions that connect the composition c_{ig} of the gas phase and the composition $c_{i \text{ liq}}$ of the liquid phase with the average composition c_{i0} in a system in which the gas and liquid phases coexist in thermodynamic equilibrium at the corresponding pressures p_g and p_{liq} . Let W be the mole fraction of the liquid phase in this system, and then

$$(1 - W) c_{ig} + W c_{i \text{ liq}} = c_{i0}. \quad (6)$$

Since the right-hand sides in Eqs. (6) are constants, W is a certain function of the pressures p_g and p_{liq} : $W = W(p_g, p_{\text{liq}})$. In principle, this function can be determined if we solve Eqs. (6) together with Eq. (3). Comparing Eqs. (5) and (6) and taking into account Eq. (4), we obtain a closed system of equations to determine the unknown quantities s , p_g , and p_{liq} :

$$\begin{aligned} \frac{dp_g}{d\eta} &= (1 - W) C_g^{-1}, \\ \frac{dp_{\text{liq}}}{d\eta} &= W C_{\text{liq}}^{-1}, \end{aligned} \quad (7)$$

$$p_g - p_{\text{liq}} = p_{\text{jump}}.$$

Thus, the problem is reduced to two ordinary differential equations with coupling. To take adequate account of phase transitions, it is sufficient to have information about the phase transitions in a mixture of fixed composition c_{i0} . In order to obtain specific solutions of system (7), it is necessary to prescribe the right-hand sides of these

equations in explicit form. However, prior to obtaining an exact solution it is possible to obtain a number of rigorous statements concerning the process of filtration by analyzing the general structure of system (7).

Thus, if the liquid phase has a nonzero threshold of mobility s_* , two situations are possible: either condensate is absent ($W = 0$) and $s = 0$, or it is present ($W > 0$) and $s > s_*$. This means that during filtration retrograde condensation should occur with a jump in saturation. On the other hand, if we fix the pressure in the gas phase p_g and let the ratio μ_g/μ_{liq} approach zero, then, as follows from (7), the saturation s tends to unity. Thus, a very viscous condensate has a tendency to accumulate in the pore space near the well.

We will seek analytical solutions of system (7) for the case where $p_{jump} = 0$. Then, system (7) is reduced to a single ordinary differential equation

$$\frac{dp}{d\eta} = (1 - W) C_g^{-1} \quad (8)$$

and an equation for determining the saturation s

$$W/(1 - W) = f_{liq} w (1 - s) \mu_g / (f_g s (1 - w) \mu_{liq}). \quad (9)$$

Let us adopt a linear dependence for the phase permeabilities:

$$f_{liq} = s, \quad f_g = 1 - s \quad (10)$$

and assume the viscosities μ_{liq} and μ_g to be constant. Experiments as well as calculations by semiempirical equations of state [3] show that the mole fraction of the liquid phase at a pressure below that of the start of condensation p_d is approximated well by the expression

$$W = a_1 p + \frac{a_2}{p - a_3}, \quad (11)$$

where a_1, a_2, a_3 are positive quantities that depend on T and c_{i0} . Let us also make the following assumptions, which are well confirmed in experiments: $n_{liq} = n_* = \text{const}$, $n_g = p/(k_b Z T)$, where k_b is the Boltzmann constant, Z is a dimensionless constant coefficient of the order of unity (the coefficient of supercompressibility). The density n_* can be correlated with a quantity having the dimensionality of pressure $p_* = k_b Z T n_*$, equal to 10^4 at in the order of magnitude.

When $p > p_d$, the liquid phase is absent ($W = 0$) and problem (8) has the well-known solution

$$p^2 \Big|_{\eta_1}^{\eta_2} = 2\mu k_b T Z q (\eta_2 - \eta_1). \quad (12)$$

When $p < p_d$, Eqs. (9) and (10) yield

$$s = W p p_*^{-1} (\varepsilon (1 - W) + W p p_*^{-1})^{-1},$$

where $\varepsilon = \mu_g/\mu_{liq}$. For the pressure we have the ordinary differential equation

$$\frac{dp}{d\eta} = \frac{q \mu_{liq} p_*}{n_* p} (\varepsilon (1 - W) + p p_*^{-1} W) = W_1. \quad (13)$$

Let us consider the region of flow where $\varepsilon(1 - W) \ll p p_*^{-1}$ (if such a region exists). In this region, instead of Eq. (13) we may use the following equation:

$$\frac{dp}{d\eta} = q \frac{\mu_{liq}}{n_*} W, \quad (14)$$

which can be integrated easily. In fact, we note that $W = 0$ when $p = p_d$ and when $p = p_1 < 0$. Then, from Eqs. (14) and (11) we obtain the solution in implicit form:

$$\frac{a_3 - p_1}{p_d - p_1} \ln \frac{p - p_1}{p_w - p_1} - \frac{a_3 - p_d}{p_d - p_1} \ln \frac{p - p_1}{p_w - p_d} = \frac{q \mu_{\text{liq}} a_1}{n_*} \eta, \quad (15)$$

$$p_w = p \Big|_{\eta=0}.$$

Relation (15) can be used to describe the distribution of the pressure near a well when p is close to p_w . When p approaches p_d , the left-hand side of relation (15) tends to $-\infty$ and therefore this relation is unsuitable. We return now to the initial equation (13).

Analysis shows that the equation $W_1 = 0$ can have: 1) three different roots p_1, p_2, p_3 , with $p_1 > p_d$; 2) three roots p_1, p_2, p_3 with $p_1 > p_d, p_2 = p_3 < 0$. In the first case the following equality is valid:

$$p(p - a_3)(p - p_1)^{-1}(p - p_2)^{-1}(p - p_3)^{-1} = \sum_{i=1}^3 A_i (p - p_i)^{-1},$$

where A_i are certain coefficients. From this equality we derive an implicit solution for p :

$$\sum_{i=1}^3 A_i \ln \frac{p - p_i}{p_w - p_i} = q \frac{\mu_{\text{liq}} a_3}{n_*} \eta. \quad (16)$$

In the second case the following identity is valid:

$$p(p - a_3)(p - p_1)^{-1}(p - p_2)^{-2} = A_1 (p - p_1)^{-1} + A_2 (p - p_2)^{-1} + A_3 (p - p_2)^{-2}$$

and, correspondingly, there is the solution

$$-A_3 (p - p_2)^{-1} + \sum_{i=1}^2 A_i \ln \frac{p - p_i}{p_w - p_i} = \frac{q \mu_{\text{liq}} a_3}{n_*} \eta - A_3 (p_w - p_2)^{-1}. \quad (17)$$

Solutions (16) and (17) join with solution (12) at the point $p = p_d$.

Thus, a solution has been found for the problem of the operation of an exploratory well of a gas-condensate field when the pressure at the face is lower than that at the start of condensation.

NOTATION

N , number of mixture components; n_{ig} and c_{ig} , density and concentration of i -th component molecules in the gas phase; n_{liq} and c_{liq} , density and concentration of i -th component molecules in the liquid phase; w , W , mole fraction of molecules in the liquid phase; s , saturation of the pore space with the liquid phase; r , distance to the well axis; r_w , well radius along the drill bit; k , permeability; h , layer thickness; μ_g , gas viscosity; μ_{liq} , condensate viscosity; T , temperature; k_b , Boltzmann constant; Z , coefficient of supercompressibility; f_g and f_{liq} , phase permeabilities of gas and liquid, respectively; p_g and p_{liq} , pressure in gas and liquid, respectively; p_{jump} , capillary jump of the pressure; p_d , pressure of the start of condensation; c_{i0} , composition of the bed mixture; $a_2, a_3, p_1, p_2, p_3, p_*$, parameters with pressure dimensionality; a_1 , dimensionless parameter; $\varepsilon = \mu_g / \mu_{liq}$, viscosity ratio; p_w , pressure in the well.

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